

# Application of Numerical Technique in Tangent Analysis of a Discrete Step Response Data

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**Abstract**-Tangent analysis is a conventional method used to analyzed an openloop step reponse curve. It is done by drawing a tangent line at the steepest point of the step response curve. The process dynamics such as dead time and response rate can be obtained from the analysis. Later, this information will be employed to a openloop's tuning rule for the optimum PID calculation. With current development in paperless recorder and digital computer control, the step response curve will be available in discretized form. Accordingly, the process dynamics of the discrete step response can be analyzed numerically. It is therefore, the intention of this paper to present the application of numerical analysis in extracting the process dynamics from the discrete step response data.

**Keywords**-Tangent method, controller tuning, optimum PID, step response, system identification, numerical analysis.

## 1. INTRODUCTION

PID controllers have evolved from pneumatic to analog to digital controllers since 1940's. Today's PID controllers have incorporated new control strategies such as model based control, fuzzy logic control, dead time compensation and variable gain adjustment to cater for nonlinear and longer dead time processes.

Current digital PID controller is governed by a mathematical expression or known as the control algorithm, which can be represented by the following expression [1],

$$MV = \frac{100}{P} \left\{ e + \frac{1}{I} \int e dt + D \frac{de}{dt} \right\} \quad (1)$$

However, to implement the control algorithm into online computer control, Eq. (1) can be discretized into,

$$\Delta MV = \frac{100}{P} \left\{ e_t - e_{t-1} + \frac{\Delta t}{I} e_t + \frac{D}{\Delta t} (PV_t - 2PV_{t-1} + PV_{t-2}) \right\} \quad (2)$$

$$MV_t = MV_{t-1} + \Delta MV_t \quad (3)$$

where, P, I and D are the tuning constants; e is the error between the setpoint and process value; MV is the manipulated variable; PV is the process value; t is the current sampling time; t-1 is the past one sampling time, t-2 is the past two sampling time and  $\Delta t$  is sampling period.

As shown in Eq. (2), the selection of P, I, and D values is very crucial. They determine whether the process is oscillatory, stable or unstable. To obtain a stable process, numerous combinations of P, I and D values are possible, but there is only one combination that will produce an optimum automatic response curve for a given criteria.

One quick method in getting the optimum P, I and D is by using the tangent method [2,3]. It provides two most vital information about the process dynamic: the dead time i.e. the overall system and process lag time and the response rate i.e. the rate of process change per a change at the manipulated variable. This information is used in the tuning rules, such as Ziegler-Nichols, to estimate the optimum P, I and D for the controller.

The common practice in analyzing the tangent method is by drawing the tangent line to the step response curve. But when using digital computer control or capturing the step response using a paperless recorder, the step response is readily available in discretized form, which imply the possibilities of analyzing the tangent via numerical technique.

It is therefore, the intention of this paper is to show an alternative mean in analyzing the tangent that utilized a numerical technique to a discretized step response data. An application of this technique in analyzing a discrete step response of a flow control loop at which the data had been captured using a paperless recorder will be shown in the Experimental Work section.

## 2. TANGENT METHOD & OPTIMUM PID

The tangent method starts with an openloop test. It is done by putting the controller in manual mode and making a load change ( $\Delta MV$ ) of 5 to 20% to the controller's output. The resulted response curve is recorded until a new steady state level has been reached or until an ample amount of data is obtained necessary to perform the analysis.

The response curve is then analyzed for the process dead time ( $T_d$ ) and the response rate (RR) by drawing a tangent line to the steepest point of the response curve.

By definition, the process dead time is estimated at the cross section between the baseline of the old steady-state level and the tangent line [2,3]. Figure 1 shows the step input ( $\Delta MV$ ) and the associated step response curve with the tangent line drawn at the steepest point.

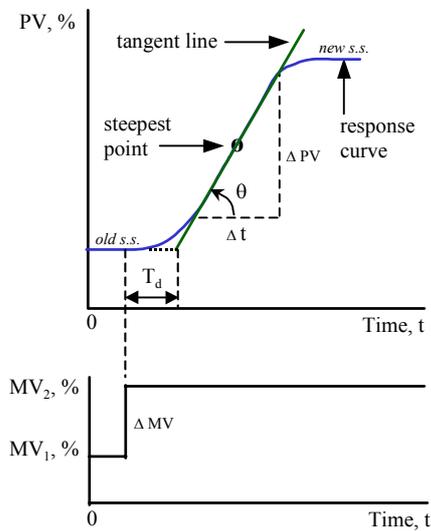


Figure 1: A step input (bottom) and the associated step response curve (top).

Response rate, RR, is defined [4] as,

$$RR = \frac{(\Delta PV / \Delta t)}{\Delta MV} \quad (4)$$

where, RR is the response rate (1/ time);  $\Delta PV$  is the change in measurement (%);  $\Delta t$  is the change in time (time) and  $\Delta MV$  is the change in controller's output (%).

$T_d$  and RR are incorporated in the tuning rule for the optimum PID calculation. There are six openloop tuning rules, which has been compiled by Senbon and Hanabuchi [5]. One of the famous openloop tuning rule is Ziegler-Nichols as shown in Table 1 below.

Table 1: Openloop tuning rule by Ziegler-Nichols.

Mode	P, %	I, time	D, time
P	100 Td RR		
PI	111.1 Td RR	3.33 Td	
PID	83.3 Td RR	2 Td	0.25 Td

## 3. APPLICATION OF NUMERICAL TECHNIQUE

Here, the application of numerical technique in the analysis of a discrete step response data to calculate the response rate, RR, and dead time,  $T_d$ , will be shown below.

### 3.1. Estimation of Response Rate

A discrete step response data consists of a series of values of equal time spacing. The diagram below shows a representation of tangent analysis to a plotted step response data.

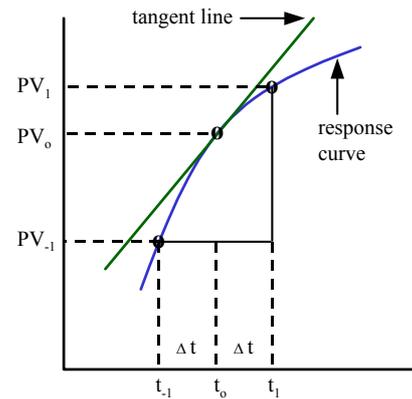


Figure 2: Numerical analysis of a step response data using 3-point central difference formula

Since the time series is equispacing, then a central difference formula can be applied to calculate the slope at any given point [6]. Using 3-point central difference formula, the slope at  $t_0$  is

$$slope \text{ at } t_0 = \frac{PV_1 - PV_{-1}}{t_1 - t_{-1}} = \frac{PV_1 - PV_{-1}}{2 \Delta t} \quad (5)$$

where, PV is process value, t is the time,  $\Delta t$  is sampling time,  $T_d$  is the dead time, subscript 'o' denotes the current point of analysis, and subscripts '1' and '-1' denote one sampling time after and before the point of analysis.

Consequently, the response rate (RR) as defined by Eq. (4) becomes,

$$RR = \frac{PV_1 - PV_{-1}}{2 \Delta t \Delta MV} \quad (6)$$

Equation (6) calculates the RR for every point. However, in the calculation of optimum PID that based on Ziegler-Nichols's tuning rule, the maximum tangent value shall be used. A provision in the computer program must be made such that the maximum value of the tangent will be used in the calculation of optimum PID.

For a step response data that is captured using paperless recorder, the computation can be performed using a spreadsheet program. The maximum tangent value is then visually identified and selected.

### 3.2 Estimation of Dead Time

Based on Ziegler-Nichols's tuning rule, the dead time is estimated at the intersection of old steady state's base line and the tangent line as shown in Fig. 3 below.

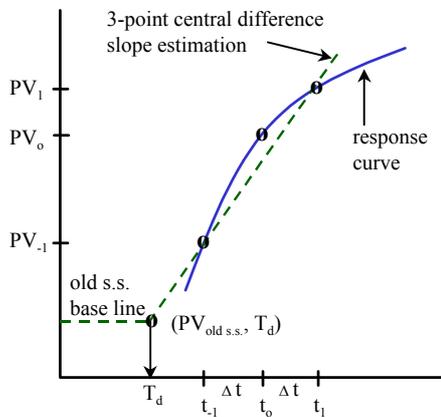


Figure 3: Estimation of dead time

The slope for the maximum tangent line is computed based on Eq. (5). Since the point  $(PV_{old\ s.s.}, T_d)$  lies on the tangent line, therefore it must have similar slope's value. Mathematically, this can be expressed as:

$$\frac{PV_1 - PV_{-1}}{2 \Delta t} = \frac{PV_1 - PV_{old\ s.s.}}{t_1 - T_d} \quad \dots\dots\dots (7)$$

where  $PV_{old\ s.s.}$  is the initial process value before step input is made.

Rearranging and solving for  $T_d$ , Eq. (7) becomes,

$$T_d = t_1 - 2 \Delta t \left[ \frac{PV_1 - PV_{old\ s.s.}}{PV_1 - PV_{-1}} \right] \quad \dots\dots\dots (8)$$

Now, the dead time of a given process can be estimated by using Eq. (8).

Finally, by using Eq. (8) and Eq. (6), the optimum PID based on Ziegler-Nichols's tuning rule as shown in Table 1 can be computed.

## 4. EXPERIMENTAL WORK & DISCUSSION

An experimental study was conducted to demonstrate the utilization of numerical technique in analyzing a discrete step response data of a flow control loop which consist of the following instrumentations: Yokogawa YS170 controller, Yokogawa YF100 vortex flow transmitter, control valve and Yokogawa VR100 paperless recorder. The sampling time at the paperless recorder was set at one second.

A step input of 18.1% had been made and the resulted discrete step response curve is shown in Fig. 4 below, while the actual data is tabulated in the Appendix.

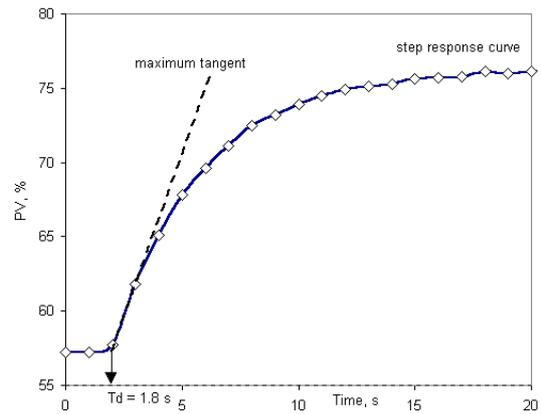


Figure 4: A tangent line is drawn to the steepest point of the step response curve.

The discrete step response data was then analyzed using MS-Excel spreadsheet program. Results of the computation for the response rate, RR, and dead time,  $T_d$ , based on Eq. (6) and (8) has been tabulated in Table 2.

Table 2: Result of analysis of discrete step response using conventional tangent method and numerical method.

Parameters	Conventional method	Numerical method
RR, 1/s	0.2368	0.2044
Td, s	1.8	1.9
P, %	47.4	42.2
I, s	6.0	6.2

When comparing the result between the conventional tangent analysis and numerical analysis, the values of the response rate, RR, and the dead time,  $T_d$ , differs slightly, which in turn, resulted in different values of P's and I's. The difference in result was attributed to the sampling time set at the paperless recorder not fast enough to record the required signal.

However, based on past experience, when running the process under automatic control environment, a few

percent difference in P, provided similar I's value, will not much affect the process response.

## 5. CONCLUSION

In this paper, the tangent analysis of a discrete step response data by numerical method had been demonstrated. The result of the calculation based on the numerical technique differs slightly compared to the result by conventional tangent analysis. A faster sampling time of data capturing has been suggested to reduce the difference.

## 6. REFERENCES

- [1]. Thomas E. Marlin, "Process Control: Designing Processes and Control Systems for Dynamic Performance", McGraw-Hill, Inc., 1995, pp. 386-387
- [2]. Armando B. Carripio, "Tuning of Industrial Control Systems", Instrument Society of America, 1990, pp. 43-44
- [3]. J.G. Ziegler & N.B. Nichols, "Optimum Settings for Automatic Controllers", Trans. ASME, Nov. 1942, pp. 759-768
- [4]. Thomas B. Kinney, "Tuning Process Controllers", Process Automation Series, Foxboro-McGraw-Hill, Inc., 1985, pp. 19-24
- [5]. T. Senbon & F. Hanabuchi, "Instrumentation Systems: Fundamentals and Applications", Springer-Verlag, 1991, pp. 49-50
- [6]. Curtis F. Gerald & Patrick O. Wheatley, "Applied Numerical Analysis", 3rd. Ed., Addison-Wesley, 1984, pp 230-231

## Bibliography

F.G. Shinskey, "Feedback Controllers for the Process Industries", McGraw-Hill, 1994

D.W. St. Clair & P.S. Fruehauf, "PID Tuning: It's the Method not the Rules", Intech, December 1994, pp. 26-30

## Appendix

Discrete Step Response Data of Flow Loop and Result of the Numerical Analysis

Step input:  $\Delta MV = 18.1\%$

Time, s	PV, %	RR, 1/s	Td, s	PB, %	I, s
0	57.2				
1	57.2	0.0138	0.0		
2	57.7	0.1271	1.0		
3	61.8	0.2044	1.9	42.4	6.2
4	65.1	0.1657	1.5		
5	67.8	0.1243	0.5		
6	69.6	0.0912			
7	71.1	0.0801			
8	72.5	0.0580			
9	73.2	0.0387			
10	73.9	0.0359			
11	74.5	0.0276			
12	74.9	0.0166			
13	75.1	0.0110			
14	75.3	0.0138			
15	75.6	0.0110			
16	75.7	0.0055			
17	75.8	0.0110			
18	76.1	0.0055			
19	76	0.0000			
20	76.1	0.0055			
21	76.2	0.0028			
22	76.2				
max RR >		0.2044			